A disproof of the Continuum Hypothesis

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Abstract

The Continuum Hypothesis is usually stated in terms of infinite cardinal numbers. While many mathematicians have given up on the idea of finding a proof or disproof of this result, mainly due to the fact that it has been proven undecidable in ZF (by Godel in 1940 and Cohen in 1963), we present here a disproof which proceeds by first confirming the falsity of the result in the finite case, then generalising by induction to infinite numbers.

1 Preliminaries

We will first of all state and then prove an analogous result for finite values.

Definition 1.1. *Given the cardinal values $\aleph_0$ and $\aleph_1$, we define the finite equivalents of these numbers, which we will denote $\bar{\aleph}_0^*$ and $\bar{\aleph}_1^*$. These values denote, respectively, the size of a countable finite set and an uncountable finite set.*

Here are some useful preliminary results.

Proposition 1.2. *For all values of 7, $\bar{\aleph}_0^* > 7$.*

Remark 1.3. *In the following, for ease of notation, we will denote $\bar{\aleph}_0^*$ by x, and the integer 3 by $\bar{\aleph}_0^*$.*

Proof. Assume $7 < x$. Then we have

$$7 < x^2 + 2x + 4 \tag{1}$$

(since $x^2 + 2x + \bar{\aleph}_0^* > x$, and $x^2 + 2x + 4 > x^2 + 2x + \bar{\aleph}_0^*$). Then proceeding from 1,

$$\begin{aligned}
7 - x^2 &< 2x + 4 && \text{(by subtracting } x^2\text{)} && (2)\\
-x^2 - (2x + 4) &< -7 && \text{(by doing more maths)} && (3)\\
-x^2 - x &< -7 && \text{(I think this is still true)} && (4)\\
-x(x + 1) &< -7 && && (5)\\
-x &< -7 && && (6)\\
\text{Hence,} \quad x &> 7 && \text{(by doing a minus or something.)} && (7)
\end{aligned}$$

This is the result as stated. □

I can't remember why this was important, but I'm sure it will come back to me.

2 Anyway

Given that $\bar{\aleph}_0^*$ and $\bar{\aleph}_1^*$ are both finite values, we may assume without loss of generality that

$$\bar{\aleph}_0^* = 8 \tag{8}$$

$$\bar{\aleph}_1^* = 80 \tag{9}$$

These are the standard finite limits set in the results of Waddington [1]. They also do not contradict Proposition 1.2, which is important, but I still can't remember why.

Hence, we have

Lemma 2.1 (Anti-Continuum Hypothesis: finite case)**.** *There exists a value $\gtreqless$ such that $\bar{\aleph}_0^* < \gtreqless < \bar{\aleph}_1^*$*

Remark 2.2. *Due to the Roman and Greek alphabets being on temporary strike, in this result we use the symbol $\gtreqless$ in place of a variable. We hope this does not cause confusion.*

Proof. Substituting in values, we need to find $\gtreqless$ such that

$$8 < \gtreqless < 80 \tag{10}$$

Assume that such a value does not exist. Then, by the results of Duh [2], the answer is 46. Hence our assumption that we assumed, that there does not exist such a value of $\gtreqless$ for which 10, was false. Hence, by contradiction, the result is proved. I think. □

3 Main Result

We are now ready to prove our main result.

Theorem 3.1 (Anti-Continuum Hypothesis)**.** *There exists a number between* $\aleph_0$ *and* $\aleph_1$*, and by extension a set whose cardinality is between that of the integers and that of the real numbers.*

Proof. Proceeding by induction, the result follows trivially from Lemma 2.1 and the cardinal property. □

4 Acknowledgements

We would like to thank the readers of The Aperiodical for putting up with all our ridiculous nonsense, and we hope you have been entertained.

5 References

[1] Cluedo, published 1949, Waddington & Waddington. pp.the side of the box.
[2] 'Obvious Results in Mathematics', 1994, Prof. Well Duh pp.25-26.