A simple proof that π is rational*

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Abstract

The number pi, written using the symbol π, is a mathematical constant that is the ratio of a circle's circumference to its diameter, and has been claimed since antiquity to be an irrational number, meaning that it cannot be expressed exactly as a ratio of two integers, and that therefore its decimal expansion never ends or settles into a permanent repeating pattern. Here a proof is given that π can indeed be expressed as a ratio of two integers, $\frac{4}{17}$, a fact that has unbelievably been overlooked until now. Moreover, this proof is understandable to anyone with a basic knowledge of algebra and calculus and arises from simply considering a standard integral at two values of x, $x = \frac{1}{4}$ and $x = 1$. Of course I doubted the result at first, given that it has been overlooked for so many years, but I have checked the proof and verified it to be correct. This is a crucial and important revelation that will significantly alter all of mathematics.

1 A formula for π

First we note that

$$\int \frac{1}{x^2 + a^2}\, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

and

$$\tan^{-1}(1) = \frac{\pi}{4}.$$

Since x can take any value, consider what happens when $x = \frac{1}{4}$. Then:

$$\int \frac{1}{x^2 + \left(\frac{1}{4}\right)^2}\, dx = 4 \tan^{-1}(1) = \pi. \tag{1}$$

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2 Consider $\frac{1}{x^2+\left(\frac{1}{4}\right)^2}$

If $x = \frac{1}{4}$, then

$$\begin{aligned}\frac{1}{x^2+\left(\frac{1}{4}\right)^2} &= \frac{1}{\left(\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)^2}\\ &= \frac{1}{\frac{1}{16}+\frac{1}{16}}\\ &= \frac{1}{\frac{2}{16}}\\ &= \frac{16}{2}\\ &= 8.\end{aligned}$$

Then

$$\begin{aligned}1 &= 8\left(x^2+\left(\frac{1}{4}\right)^2\right)\\ 1 &= 8x^2+\frac{1}{2}\\ 8x^2 &= \frac{1}{2}.\end{aligned}$$

Now, note that x^2 is equal to $x \times x$, that is x added to itself x times:

$$x^2 = x + x + \ldots + x \text{ } (x \text{ times}).$$

Then

$$8x^2 = 8(x + x + \ldots + x).$$

Multiply each side by x:

$$8x^2 = 8x(x + x + \ldots + x).$$

Now, take the derivative of each side. The left hand side is trivial, since

$$\frac{d}{dx}\left(8x^2\right) = 16x. \tag{2}$$

The right hand side requires the product rule. First let

$$u = 8x \text{ \& } v = x + x + \ldots + x.$$

Then

$$u' = 8 \text{ \& } v' = 1 + 1 + \ldots + 1.$$

Note that v' is 1 added to itself x times, i.e. $v' = x$, and v is x added to itself x times, i.e. $v = x^2$. Then the product rule gives:

$$\begin{aligned} uv' + u'v &= 8x \times x + 8 \times x^2 \\ &= 8x^2 + 8x^2 \\ &= 16x^2 \end{aligned} \tag{3}$$

Since we have, from (2) and (3), that

$$16x = 16x^2$$

it follows that $x = \frac{16}{16} = 1$.

Putting $x = 1$ into $\frac{1}{x^2+\left(\frac{1}{4}\right)^2}$, we have

$$\begin{aligned} \frac{1}{1^2 + \left(\frac{1}{4}\right)^2} &= \frac{1}{1 + \frac{1}{16}} \\ &= \frac{1}{\frac{16}{16} + \frac{1}{16}} \\ &= \frac{1}{\frac{17}{16}} \\ &= \frac{16}{17}. \end{aligned} \tag{4}$$

3 The final step

It follows from (4) that

$$\begin{aligned} \int \frac{1}{1^2 + \left(\frac{1}{4}\right)^2}\, dx &= \int \frac{16}{17}\, dx \\ &= \frac{16}{17}x \end{aligned} \tag{5}$$

From (1), we know that this equation equals π when $x = \frac{1}{4}$, i.e.

$$\pi = \frac{16}{17} \times \frac{1}{4} = \frac{4}{17}. \tag{6}$$

Clearly, $\frac{4}{17}$ is rational, and therefore π is rational. $\square$