A disproof of the Continuum Hypothesis

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Abstract

The Continuum Hypothesis is usually stated in terms of infinite cardinal numbers. While many mathematicians have given up on the idea of finding a proof or disproof of this result, mainly due to the fact that it has been proven undecidable in ZF (by Gödel in 1940 and Cohen in 1963), we present here a disproof which proceeds by first confirming the falsity of the result in the finite case, then generalising by induction to infinite numbers.

1 Preliminaries

We will first of all state and then prove an analogous result for finite values.

Definition 1.1. Given the cardinal values $\aleph_0$ and $\aleph_1$, we define the finite equivalents of these numbers, which we will denote $\bar{\aleph}_0^*$ and $\bar{\aleph}_1^*$. These values denote, respectively, the size of a countable finite set and an uncountable finite set.

Here are some useful preliminary results.

Proposition 1.2. For all values of $7$, $\bar{\aleph}_0^* > 7$.

Remark 1.3. In the following, for ease of notation, we will denote $\bar{\aleph}_0^*$ by $x$, and the integer $3$ by $\bar{\aleph}_0^*$. 

Proof. Assume $7 < x$. Then we have

$$7 < x^2 + 2x + 4$$

(1)
(since \(x^2 + 2x + \bar{\aleph}_0^* > x\), and \(x^2 + 2x + 4 > x^2 + 2x + \bar{\aleph}_0^*\)). Then proceeding from 1,

\[
\begin{align*}
7 - x^2 &< 2x + 4 \quad \text{(by subtracting \(x^2\))} \\
-x^2 - (2x + 4) &< -7 \quad \text{(by doing more maths)} \\
-x^2 - x &< -7 \quad \text{(I think this is still true)} \\
-x(x + 1) &< -7 \\
-x &< -7 \\
\text{Hence, } x &> 7 \quad \text{(by doing a minus or something.)}
\end{align*}
\]

This is the result as stated.

I can’t remember why this was important, but I’m sure it will come back to me.

## 2 Anyway

Given that \(\bar{\aleph}_0^*\) and \(\bar{\aleph}_1^*\) are both finite values, we may assume without loss of generality that

\[
\begin{align*}
\bar{\aleph}_0^* &= 8 \\
\bar{\aleph}_1^* &= 80
\end{align*}
\]

These are the standard finite limits set in the results of Waddington [1]. They also do not contradict Proposition 1.2, which is important, but I still can’t remember why.

Hence, we have

**Lemma 2.1** (Anti-Continuum Hypothesis: finite case). *There exists a value \(\gtrless\) such that*

\[
\bar{\aleph}_0^* < \gtrless < \bar{\aleph}_1^*
\]

**Remark 2.2.** *Due to the Roman and Greek alphabets being on temporary strike, in this result we use the symbol \(\gtrless\) in place of a variable. We hope this does not cause confusion.*

**Proof.** Substituting in values, we need to find \(\gtrless\) such that

\[
8 \lesssim \gtrless \lesssim 80
\]

Assume that such a value does not exist. Then, by the results of Duh [2], the answer is 46. Hence our assumption that we assumed, that there does not exist such a value of \(\aleph\) for which 10, was false. Hence, by contradiction, the result is proved. I think.
3 Main Result

We are now ready to prove our main result.

**Theorem 3.1** (Anti-Continuum Hypothesis). *There exists a number between \( \aleph_0 \) and \( \aleph_1 \), and by extension a set whose cardinality is between that of the integers and that of the real numbers.*

*Proof.* Proceeding by induction, the result follows trivially from Lemma 2.1 and the cardinal property.

4 Acknowledgements

We would like to thank the readers of The Aperiodical for putting up with all our ridiculous nonsense, and we hope you have been entertained.

5 References